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Transition at dissipative scales in large-Reynolds-number turbulence

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Among the available diagnostics of turbulence, the flatness of the velocity derivatives is particularly interesting because it represents a straightforward test of Kolmogorov theory, and provides a quantitative estimate for intermittency effects. It is commonly considered that the flatness factor increases with the Reynolds number, following a power law at high Reynolds numbers. At variance with this picture, evidence for a transitional behavior, taking place around the Taylor microscale Reynolds number $R_\lambda = 700$, has been recently obtained in several experiments. In the present paper we study this transition in detail, and show it has the characteristics of a second order phase transition. We propose a physical picture for this transition, based on worm vortex breakdown, which leads as to suggest that intense sub-Kolmogorov structures might develop above the transition point. These results indicate that the existence of an asymptotic state at infinite Reynolds number may become questionable and more generally, that our current views on dissipative range intermittency probably need to be revised

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For 50 years, measurements have supported the idea that at high Reynolds numbers, turbulent flows develop self-similar ranges of scales across which the energy, injected at large scales, cascades towards smaller and smaller scales until it gets burned. This picture was proposed 60 years ago by Kolmogorov [1]. It received experimental support in several respects, the most famous being the omnipresence of $k^{-5/3}$ energy spectra in turbulent flows [2]. The cascade picture has been refined many times, along different lines of thought, to incorporate intermittency and anisotropy effects, but it has never been durably challenged [3].

For decades, all measurable implications of the cascade picture have been subjected to experimental check. This is the case for the Reynolds number dependence of the flatness of the longitudinal velocity derivatives, defined by

$$F = \frac{\left\langle \left(\frac{\partial u}{\partial x} \right)^4 \right\rangle}{\left\langle \left(\frac{\partial u}{\partial x} \right)^2 \right\rangle^2}, \tag{1}$$

in which u is the flow component along direction x. This factor represents a direct signature of small scale intermittency. The larger the flatness factor, the more intermittent the system. F may also be related to the fluctuations of the energy dissipation, and, under some assumptions, to the local acceleration of fluid particles. Cascade models predict the flatness grows as a power law with the Reynolds number [3]. At the moment, several collections of numerical and experimental data are available. The latest review, published in 1997, shows that F keeps increasing with the microscale Reynolds number [4]. The corresponding plot is shown in Fig. 1. It is generally considered that a power law provides an acceptable fit to the measurements, over a range of variation of the microscale Reynolds number R_λ covering two decades, between 200 and 20 000. In this respect, the law which is currently proposed [5] has the following expression:

$$F \sim R_\lambda^{0.35}.$$

R_λ is the microscale Reynolds number, a quantity based on an internal flow scale (the Taylor scale), and which, for the flows considered, is proportional to the square root of the large scale Reynolds number. The existence of a positive exponent reveals substantial deviations between Kolmogorov theory (which predicts F is a constant) and experiment, and provides experimental support to multifractal modeling [3]. Incidentally, the exponent is found incompatible with essentially all the existing structural models of turbulence, jeopardizing their relevance to the description of turbulence statistics, despite their appealing physical content.

In recent years, our group conducted a series of experiments in low temperature helium gas, in the hope of measuring, with an improved accuracy, the evolution, with the Reynolds number, of the aforementioned flatness factor [6,7]. The experiments were performed between counter-rotating disks, equipped with blades, and the measurement was made with hot wires, designed to work at low temperatures. The

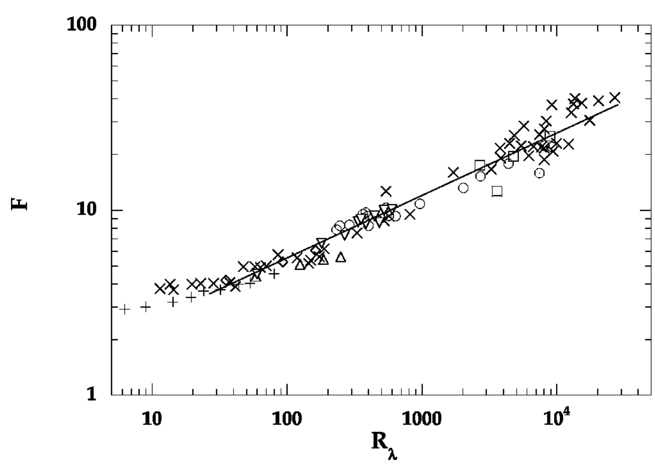

FIG. 1. Plot of the flatness versus the Reynolds number, collected by Sreenivasan and Antonia.

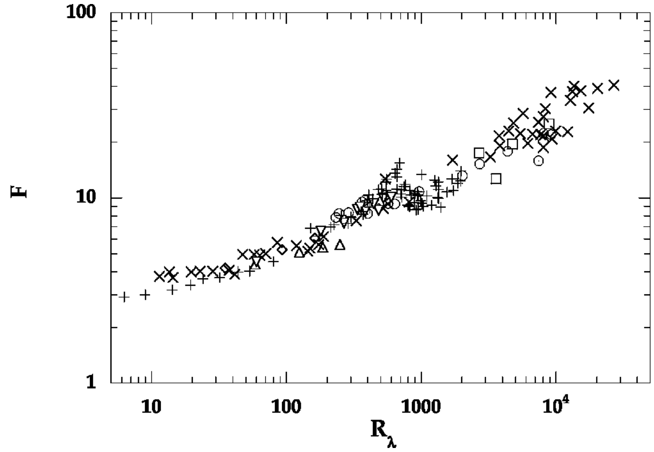

FIG. 2. Plot of the flatness versus the Reynolds number, collected by Sreenivasan and Antonia.

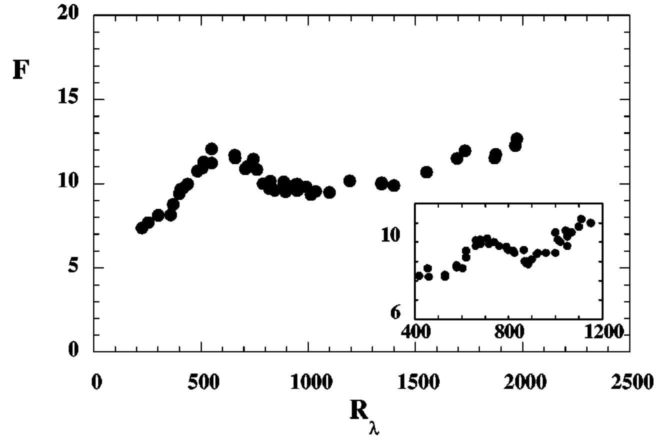

FIG. 3. Plot of the flatness of the longitudinal velocity derivatives versus the microscale Reynolds number R_λ for data set (a); the inset shows Pearson's measurements of the same quantity, confirming the transitional behavior around $R_\lambda = 700$.

probe typically resolved the Kolmogorov scale, had a time response comparable to the Kolmogorov time; moreover, the statistics was comfortably well converged. It came as a surprise that instead of a power law, we observed a transitional behavior: we found that the flatness factor first increased up to $R_\lambda = 700$, then decreased and eventually increased again (see Fig. 3). Much effort was spent to identify possible artifacts [8,9], and it was concluded that such a possibility is unlikely [10]. The effect was further confirmed on investigating the higher order moments of the velocity derivatives ([7,11]). The quality of the measurement technique leading to this observation was further exploited to check the Kolmogorov equation with unprecedented accuracy [12], within a range comfortably encompassing the transitional region. Later, by using the same apparatus, but implementing a different flow configuration, we came to suggesting that the transition is a universal phenomenon [13]. It should be noted that the existence of such a transition does not contradict previous measurements made on other systems: due to the rather large experimental uncertainty of such measurements, and the poor coverage of the range 700–1000 (see Fig. 1, it is hard to decide whether a transition takes place or not around $R_\lambda = 700$. On the other hand, if one adds our data to the compilation of Fig. 1, one sees the existence of a transitional behavior is consistent with the whole set of data (see Fig. 2).

It turns out that, in a recent study, Bruce Pearson [14] showed that a similar transition takes place in the wake produced by a porous plate. He reached this conclusion by using a classical wind tunnel, and standard hot wire anemometry. He observed that the flatness factor of the longitudinal derivatives first increases up to R_λ around 700, then decreases, and eventually increases again. The data, shown in the inset of Fig. 3, is strikingly comparable to our observations. Now we can take as a serious possibility that the transitional behavior we observed several years ago is an intrinsic property of turbulence, since at the moment it has been observed in all flow configurations where accuracy requirements could be met. Along this line of thought, the supposed established plots of these quantities will probably have to be entirely reconsidered (see Fig. 2), along with their current interpretation.

Prompted by Bruce Pearson's results, we found it useful to report a further analysis of the phenomenon. In this paper we investigate in detail the ''transitional behavior'' observed in the helium experiment; we show that the transitional behavior is not just a kink on a curve, or a crossover phenomenon, but appears to have the characteristics of a second order phase transition. We discuss a possible interpretation of this result, referring in particular to a scenario anticipated 10 years ago by Jimenez *et al.* [16], and which we offer to complete.

The data we analyze here has been obtained in low temperature helium gas, between counter-rotating disks. Some of the measurement has already been published (see [7]), but not in the form we propose here. We refer the reader to Refs. [6,7] for more detail on the experiment. We call this data set (a). In the present paper, we also incorporate a set of measurements made in slightly different conditions (we added a small grid in the flow, far away from the probe), with a

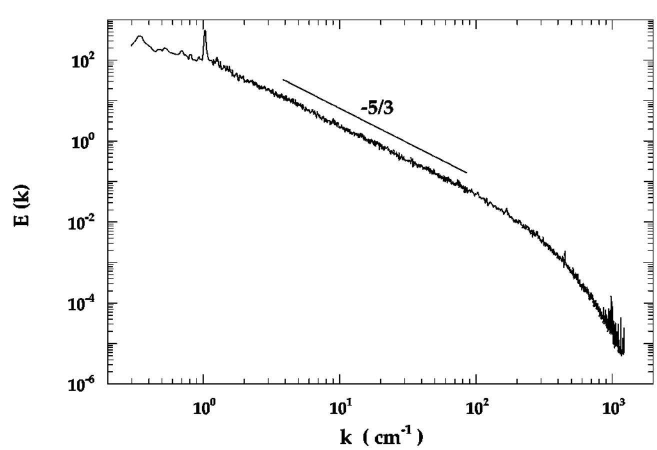

FIG. 4. Typical energy spectrum, obtained for $R_\lambda = 1350$, for set (a).

modified probe system, allowing us to obtain an improved ratio signal over noise. This set is called set (b). A typical spectrum we obtained is displayed in Fig. 4; it illustrates what we believe to be an excellent quality of measurements.

The plots showing the transitional behavior are shown in Fig. 3. The quantity we measure is the flatness of the longitudinal velocity derivative. In practice, to measure this quantity, we first determine the flatness of the velocity increment, defined by

$$F=\frac{\langle [u(x+r)-u(x)]^4\rangle}{\langle [u(x+r)-u(x)]^2\rangle^2},$$

where u is the velocity component along the mean flow, and x and r are coordinates parallel to the mean flow and the angular brackets mean time averaging. To determine the flatness of the derivative, we extrapolate $F(r)$ down to zero separations.

In Fig. 3 each point is an average between itself and the two nearest neighbors. The range of the Reynolds number was sufficiently well resolved to perform this operation, without deteriorating the resolution, in terms of R_λ. By doing so, we could reduce the scatter to a few percent, which is one order of magnitude better than previous measurements. Owing to the complexity of the evolution of the flatness factor with R_λ, proposing a fit for the experimental data is not straightforward; the simplest approach seems to assume we have a transition, and we analyze it using the usual representations of critical phenomena. Along this line of thought, a critical value for this transition and for set (a) is

$$R_{\lambda c}=650.$$

The corresponding critical flatness is $F_c=12\pm 2$.

We may further, as in critical phenomena, define two regions, below and above the postulated critical point. Below the critical point, we may fit the flatness factor by the following power law:

$$F_0(R_\lambda)=0.37R_\lambda^{0.54}.$$

A similar formula has already been proposed before ([7]), in the context of a structural interpretation of the flatness evolution. Above the transition, we may write, still in the spirit of critical phenomena, a formula for $F(R_\lambda)$:

$$F(R_\lambda)=F_0(R_\lambda)+G(R_\lambda)$$

and determine G. Function G is plotted in Figs. 5 and 6 as a function of R_λ and, on a logarithmic plot, as a function of $R_\lambda-R_{\lambda c}$, respectively. The transition is rather neat in Fig. 5. As shown in Fig. 6, one may propose the following law for G:

$$G=0.02 \quad (R_\lambda-R_{\lambda c})^{0.5}.$$

There is thus a critical exponent, which turns out to be close to 0.5 over one decade of variation of R_λ; note, however, the accuracy on the estimate of the exponent is poor. Exponents lying in the range 0.4–0.6 would be acceptable as well; the coefficient 0.02 does not provide much more than

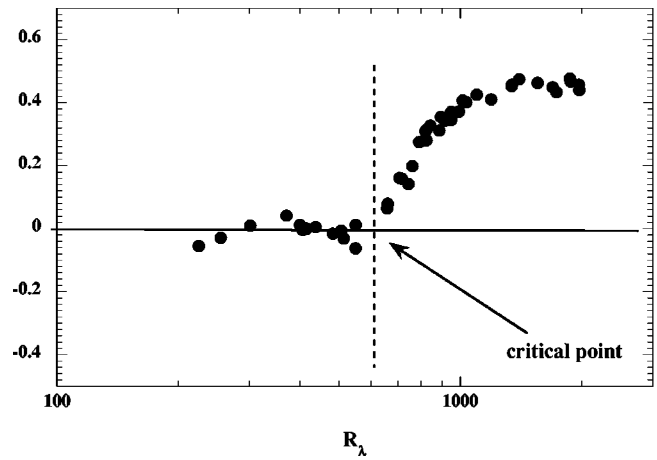

FIG. 5. Plot of function G, defined above, using semilogarithmic scales; the plot suggests a transitionlike behavior.

an order of magnitude for the prefactor. At larger R_λ, and up to the largest Reynolds number we have, it appears that function G tends to level off. Nonetheless, more data are needed to ascertain whether there is a plateau in this range of Reynolds numbers. The same analysis can be carried out for the hyperflatness factors of orders 5 and 6, leading to similar results.

In fact, the transition we display here is not limited to the dissipative range of scales; it also affects scales intermediate between the dissipative and the inertial range, in a way that dissipative and inertial ranges of scales may behave differently with respect to the Reynolds number. In particular, the width of the zone separating the dissipative from the inertial

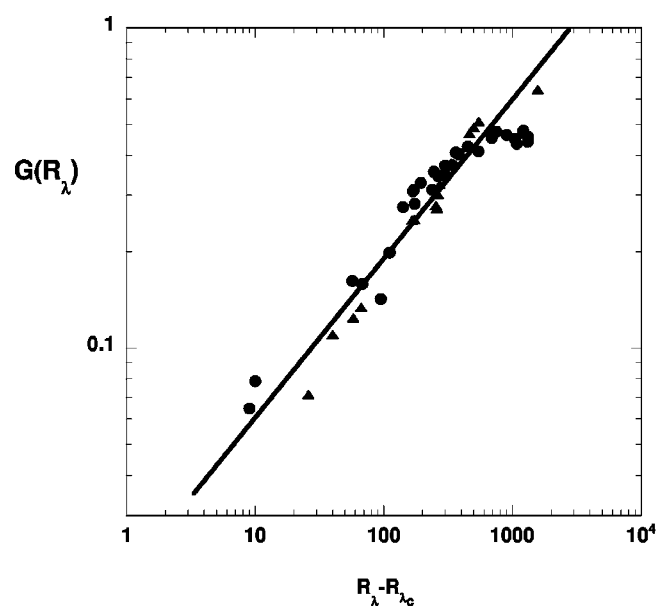

FIG. 6. Plot of function G versus the difference between the actual Reynolds number and the critical one; disks are data set (a) and triangles are data set (b). As is usually done in critical phenomena, we adjust the critical Reynolds number for each data set. For set (b), the critical point is located at $R_\lambda=670$. The full line has a slope equal to 0.5, and is shown to guide the eyes.

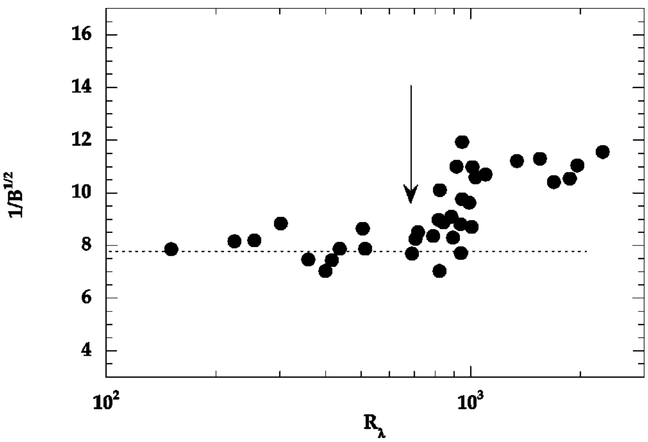

FIG. 7. Plot of the width of the region separating the dissipative from the inertial range versus the Reynolds number, measured in terms of the Kolmogorov scale η, for data set (a).

domains undergoes a transitional behavior, consistent with the one seen on the flatness measurements. To determine such a width, we use a method proposed by Stolovitsky *et al.* [18]: the structure function of order 4, defined by

$$S_4 = \langle [u(r+x) - u(x)]^4 \rangle,$$

where u is the component along the mean flow (along which the coordinates x and r are defined), and the angular brackets mean statistical averaging. Following Stolovitsky *et al.*, S_4 is fitted by using the formula

$$S_4(r) = \frac{Ar^4}{[1+(B/r)^2]^{0.7}}.$$

In this formula, η is the Kolmogorov scale and A and B are fitting parameters; the parameter $B^{-1/2}$ represents the width, in units of the Kolmogorov scale, of the region separating the dissipative from the inertial domains. The plot of $B^{-1/2}$ versus R_λ is shown in Fig. 7 for data set (a). At low R_λ, we recover previous estimates (see [18]); nonetheless, one sees a transition around $R_\lambda = 700$, whose characteristics are consistent with those discussed in the preceding section. Thus, the width of the intermediate domain of scales, separating the dissipative from the inertial ranges, also undergoes a transition, similar to the one previously described, and is located at the same position on the R_λ axis.

Concerning the inertial range, we did not observe any measurable transitional behavior [19]. The transition is therefore limited to the dissipative range of scales and to the intermediate region between the dissipative and inertial ranges.

We now come to the discussion of this transition. The possibility to physically understand the transition is to invoke vortex breakdown [15]. This interpretation bears on the premise that the dissipative range of scales is organized into vortex filaments (called worms), with diameters on the order of a few Kolmogorov scales, and with a velocity difference across them equal to a fraction of the standard deviation of the velocity field [16]. It is not a complicated matter to show that these worms control the fourth order moment of the velocity derivatives, while the second order moment is controlled by the background fluctuations [7]. Accordingly, the flatness factor, which is the ratio between the two, can be estimated. By assuming the worm density is a constant, one gets the result that the flatness increases as $R_\lambda^{1/2}$, which may provide an interpretation for the exponent found in the pretransitional regime. On the other hand, as the Reynolds number increases, the internal Reynolds number of the worms increases as $R_\lambda^{1/2}$ ([16]); it follows that vortex breakdown should occur in some (undetermined) upper range of the Reynolds number, as conjectured in Ref. [16]; a vortex breakdown of the worms would lead to an increase of their size, and for a probe resolving no more than the Kolmogorov scale, this would induce a reduction of the highest gradients and, in turn, a decrease of the flatness factor.

However, since the worms characteristics are broadly distributed, one could expect, upon an increase of the Reynolds number, that the decrease of the flatness factor is a progressive process, possibly masked by other effects, such as the reduction of the Kolmogorov scale which favors an intensification of the gradients; this contrasts with our experiment, and that of Bruce Pearson as well, which both indicates a sharp transition, with a possible discontinuity of the derivatives at the transition point. The sharpness of the transition may be accounted for if one assumes the worms are correlated. Such a correlation has been underlined in recent studies, indicating that worms are organized into internally coherent clusters [17]. In this context, a second order transition can be envisaged. In condensed matter words, we would shift from a paramagnetic to a ferromagnetic situation. At the moment, however, this may be taken as a possibility, which certainly requires further work to be assessed. In this respect, additional information using X wire probes, or a numerical analysis of the flow field close to the transition, may be extremely valuable.

An interesting consequence of these arguments concerns the infinite Reynolds number limit of turbulent flows: if vortex breakdown occurs, worms debris will be generated. The characteristics of these new (probably intense) structures are difficult to foresee, but one may argue that, as the Reynolds number is increased well beyond the transition point, the debris will in turn become unstable (see [20]); there is *a priori* no strong reason the sequence may stop. It is thus not ascertained we ever reach an asymptotic state, free of bifurcation at infinite Reynolds numbers, as cascade theories propose. An experimental implication of this discussion is that one must resolve the debris above $R_{\lambda c}$ to measure flatness factors; since most of the available measurements do not resolve more than one Kolmogorov scale, one may suspect the flatness factors published in Ref. [4] are substantially underestimated in the upper Reynolds number range. All flatness measurements for $R_\lambda > 1500$ should perhaps be redone using probes resolving sub-Kolmogorov scales.

If one considers the statistical approaches, one may say that, at the moment, there is no way to explain the transition; the proof being that it has never been predicted. It can perhaps be proposed that the transition is a finite Reynolds number effect which statistical theories are not supposed to address; however, there is no guarantee that other transitions do not arise at higher Reynolds numbers: as discussed above, the concept of an asymptotic state, free of transition at infinite Reynolds numbers seems at the moment questionable.

To conclude, we offer here a detailed characterization of a transition, observed around $R_\lambda = 700$, which may seriously be taken as an intrinsic property of all high Reynolds number turbulent flows . The main outcome of the analysis is that the transitional behavior has the characteristics of a second order phase transition. Confirming the nature of the transition would require additional measurements, similar to those currently carried out for critical phenomena. What we can say, at the moment, is that we assume a transition provides a simple and consistent framework for analyzing the data. Concerning the interpretation, one must warn the reader that our proposal, although physically plausible, has no theoretical background at the moment. What we propose here is to link the transition we observe to a breakdown of the worms, in a context where some internal correlation exists. Such a proposal raises several issues, in particular the nature of the turbulent state at infinite Reynolds numbers, the reliability of the flatness measurements above the transition point and the existence of sub-Kolmogorov vortex filaments; we also note that the existence of a transition at such a high Reynolds number fits well with a structural approach to turbulence, but somehow challenges the statistical view, which so far is considered as the most appropriate road to ''solving'' the turbulence problem.

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[11] The review of Ref. [4] considers as an established fact that the hyperflatnesses of orders 5 and 6 monotically increase with the Reynolds number, referring to a study by Antonia *et al.* [Boundary-Layer Meteorol. **21**, 15 (1981)] to support this statement. Data in the above reference are scarce, and it seems difficult to use this paper to draw out any accurate conclusion about the evolution of the aforementioned factors. Our hyperflatness measurements, published in Ref. [7] are much more detailed, and clearly shows a transitional behavior.

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[20] A tentative way to estimate the characteristics of the debris would be to assume they have the form of vortex filaments, immerged in a background field of intensity u' and subjected to a strain, locally generated by the worm, on the order of u'/η. In such a situation, the limiting width of these subfilaments would be on the order of [$\nu\eta/u' \sim LR_\lambda^{-7/4}$ (in which ν is the kinematic viscosity, η is the Kolmogorov scale, and L is the large scale)], and the corresponding internal Reynolds number would be on the order of $R_\lambda^{1/4}$; the debris thus define intense subkolmogorovian structures. Because their internal Reynolds number increases with R_λ, these subfilaments are in turn expected to burst as R_λ is further increased. One may iterate the argument for the next generations of debris; the reasoning leads to defining a hierarchy of subkolmogorovian structures, which successively become unstable as R_λ is increased. A similar hierarchy was obtained by [16]. However, in their paper the authors conclude the system ultimately becomes stable at infinite Reynolds numbers.